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Calculus BC
2022-2023

Document: APCalculusBCSyllabusDocument.odt

Curricular Requirements (College Board)[1]

No.	Requirement	Applicable portions of Syllabus
CR1	The students and teacher have access to a college-level calculus textbook, in print or electronic form.	Course Materials, p 5 (Primary Textbook is Calculus Graphical, Numerical Algebraic, Finney et al., 3rd Ed.)
CR2	The course is structured to incorporate the big ideas and required content outlined in each of the units described in the AP Course and Exam Description.	*Course Outline, p8. Our primary text (Finney) tackles some topics in different order (particularly series and advanced treatment of parametric equations) than the AP(r) Course and Exam Description, sometimes uses slightly different nomenclature, often combines many required sub-units into one section, and in a few cases is inordinately light or absent in treatment of required subunits. A very careful section-by-section and often page-by-page, and example-by-example comparison of the Finney 3rd Ed. text with the required content of each of the units of the AP(r) Course and Exam Description has revealed which subunits require additional explanation, emphasis, or supplementation from our additional text and instructor resources. With these adjustments, our course is able to incorporate all the required content outlined in each of the units described in the AP Course and Exam description and the Big Ideas.*
CR3	The course provides opportunities for students to develop the skills related to Mathematical Practice 1: Implementing Mathematical	See specific subsection Curricular Requirement 3, p21, below for two examples, and see Course Outline where

1 These requirements are explicitly listed in the College Board document publicly available at: https://apcentral.collegeboard.org/media/pdf/ap-calculus-bc-syllabus-development-guide-2021_1.pdf

No.	Requirement	Applicable portions of Syllabus
	Processes. • 1A Identify the question to be answered or problem to be solved. • 1B Identify key and relevant information to answer a question or solve a problem • 1C Identify an appropriate mathematical rule or procedure based on the classification of a given expression. (e.g., Use the chain rule to find the derivative of a composite function). • 1D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g. rate of change and accumulation) or processes (e.g., differentiation and its increase process, anti-differentiation) to solve problems. • 1E Apply appropriate mathematical rules or processes, with and without technology • 1F Explain how an approximated value relates to the actual value	Mathematical Practice 1 is included in multiple sections.
CR4	The course provides opportunities for students to develop the skills related to Mathematical Practice 2: Connecting Representations. • 2A Identify common underlying structures in problems involving different contextual situations. • 2B Identify mathematical information from graphical numerical, analytical and/or verbal representations • 2C Identify a re-expression of mathematical information presented in a given representation. • 2D Identify how mathematical characteristics or properties of function are related in different representations	See specific subsection <u>Curricular Requirement 4</u>, p 23, below for three examples, and see Course Outline where Mathematical Practice 2 is included in multiple sections.

No.	Requirement	Applicable portions of Syllabus
	• 2E Describe the relationships among different representations of functions and their derivatives.	
CR5	The course provides opportunities for students to develop the skills related to Mathematical Practice 3: Justification. • 3A Apply technology to develop claims and conjectures. • 3B Identify an appropriate mathematical definition theory, or test to apply. • 3C Confirm whether hypotheses or conditions of a selected definition, theory or test have been satisfied. • 3D Apply an appropriate mathematical definition theorem, or test. • 3E Provide reasons or rationales for solutions and conclusions. • 3F Explain the meaning of mathematical solutions in context. • 3G Confirm that solutions are accurate and appropriate.	See specific subsection Curricular Requirement 5, p 25, below for two examples, and see Course Outline where Mathematical Practice 3 is included in multiple sections.
CR6	The course provides opportunities for students to develop the skills related to Mathematical Practice 4: Communication and Notation. • 4A. Use precise mathematical language • 4B Use appropriate units of measure • 4C Use appropriate mathematical symbols and notation (e.g. represent a derivative using f'(x), y', and dy/dx) • 4D Use appropriate graphic techniques • 4E Apply appropriate rounding procedures	See specific subsection Curricular Requirement 6, p27, below for two examples, and see Course Outline where Mathematical Practice 4 is included in multiple sections.

No.	Requirement	Applicable portions of Syllabus
CR7	Students have access to graphing calculators and opportunities to use them to solve problems and to explore and interpret calculus concepts.	See specific subsection Curricular Requirement 7, p29, below for both access and specific examples of technology application, and see Course Materials where the provision of a graphing calculus is again repeated.
CR8	The course provides opportunities for students to use calculus to solve real-world problems.	See specific subsection Curricular Requirement 8, p31, below for two examples

COURSE MATERIALS[2] (CR1)

PRIMARY TEXTBOOK
Prentice Hall. Finney RL, Demana FD, Waits BK, Kennedy D. Calculus Graphical, Numerical, Algebraic. 3rd Ed. Pearson Prentice Hall, Copyright 2007.

Review Reading and Additional Problems:
The Princeton Review. Princeton Review AP Calculus BC Prep 2023 (or 2021, or 2022) copyright 2023 (or 2021 or 2022) -- provides sample problems from past tests and a different type of presentation of material compared to Finney, and for a few topics, provides material not well covered by Finney.

TECHNOLOGY
All students are expected to have a TI-84+CE calculator for their use in class and for homework assignments. For students that cannot afford this calculator, our school will loan a calculator to that student for the course.

The graphical calculator will be used very frequently in our class **(CR7)** and we will mentor students to help them become more adept in the use of this technology to help them understand the various concepts and to more efficiently process problems. Additionally, we will provide them with a basic understanding of how to create simple programs to carry out repetitive processes such as Riemann sums.

Throughout the course, students will be instructed in at least four functionalities allowed on the AP Exam with the graphical calculator including

- Plotting the graph of a function within an arbitrary viewing window
- Finding the zeros of a function (solving equations numerically)
- Numerically calculating the derivative of a function (nDeriv)
- Numerically calculating the value of a definite integral (fnInt)

THE INTERNET
Students will find innumerable resources available to them from the Internet.

2 The text by Finney meets the level of college-level Calculus. However its coverage of required subtopics 6.10 & 6.12 is inadequate and thus will be supplemented.

ASSESSMENT

This is an A/P Course. By definition, it is taught, tested, and graded at the College level. The literature suggests that students are most benefited when a high-school calculus course is not at all watered down from a college-level course. As much as possible, tests will be drawn from past AP tests. The exact relationship between raw scores on true AP-level questions, and applicable -------- grades is subject to adjustment, but begins roughly at this level:

Raw Score	Scaled Score
75%	90% = A
65%	80% = B
45%	70% = C

These levels approximate the "5" "4" "3" levels of AP tests.

All students are required to take the AP Calculus BC test, or in special circumstances, the AB test. This helps validate the integrity of our course structure.

Students that consistently score lower than what is appropriate for an A/P Score of 4, should expect to have special discussions with their parents and the Instructor to decide how to proceed.

Homework	25%
Tests and Laboratories *There is a possibility of unannounced quizzes at any time, which will be given the weighting of 1/2 a test, presuming I can make a way to make that happen.*	50%
Final Exam (at the end of each Semester)	25%

Classroom Behavior
I do not anticipate having any difficulties with classroom behavior. The class will be taught as a college level class, with rigorous lecture, debate, and probing questions.

Make-up Policy
Excused absence/s due to sickness will merit the Make-up Policy. One day of extension is given for each day of absence.

Students who have a scheduled trip or a planned absence are expected to submit completed work upon return to class. This is also true to quizzes or tests. Participation in a sport activity is proof of

ATTENDANCE of that day of school per school rules, and therefore does not excuse a student for their responsibilities toward the PRIMARY goal of Christian Education: which is Education.

Early notice and arrangement should be made for convenience and order.

Late Work

- An assignment or homework is to be turned in at the class period and time designated by the teacher, typically at the beginning of the period. Teachers are to designate the venue for receiving the assignment or homework, electronic, hard copy, or other.
- Work not turned in as the manner delineated above will be late. The table below lists the points to be deducted per day late.

Days Late	One Day	Two Days	Three Days	Four or More
Logic & Rhetoric	- 11 percent	- 21 percent	- 31 percent	Not Accepted
	LATE homework cannot be guaranteed to be graded to the same standard as homework turned in on the appropriate day, and cannot be guaranteed to be graded or returned in the same timely fashion as appropriately completed work.			

- As an example, Rhetoric homework assignment turned in one day late, and receiving a grade of 80% will then be reduced to 69% (reduced by 11 percent) for being turned in late.
- There are two exceptions to this standard:
  - If a student has an unplanned, but excused, absence, the due date will be extended by the number of days the student was absent.

COURSE OUTLINE

The ------------- AP Calculus BC course is a college level Calculus class including limits, derivatives, integral calculus and sequences and series. It is made available to selected students with appropriate mathematical backgrounds. Our class periods are the normal schedule at our school. Additional instructional time by appointment as needed. Because of our very small class size, we can be more flexible.

Our Calculus Course provides in depth understanding of Calculus, and naturally includes the following Big Ideas:

BIG IDEA 1: CHANGE (**CHA**) Using derivatives to describe rates of change of one variable with respect to another or using definite integrals to describe the net change in one variable over an interval of another allows students to understand change in a variety of contexts. It is critical that students grasp the relationship between integration and differentiation as expressed in the Fundamental Theorem of Calculus—a central idea in AP Calculus.

BIG IDEA 2: LIMITS (**LIM**) Beginning with a discrete model and then considering the consequences of a limiting case allows us to model real-world behavior and to discover and understand important ideas, definitions, formulas, and theorems in calculus: for example, continuity, differentiation, integration, and series BC only.

BIG IDEA 3: ANALYSIS OF FUNCTIONS (**FUN**) Calculus allows us to analyze the behaviors of functions by relating limits to differentiation, integration, and infinite series and relating each of these concepts to the others.

UNIT AND SUB UNIT CONTENT

Our -------------AP Calculus class **teaching content** meets each and every one of the Unit and Sub Unit content requirements, presented in the AP(R) Calculus AB and BC Course and Exam Description, with occasional additional material and/or additional emphasis on specific items, as shown below in TABLE ONE. This table was created by a careful, section by section, and at times page-by-page analysis of the AP requirements with the content of the primary textbook.

TABLE ONE: Comparison of AP(R) Unit/Sub Unit Content with Course Sections

AP Syllabus Requirements	Covered in Finney Section	Additional Treatment Provided to Sharpen Focus
(Not required)	Chapter 1	Brief review as needed to shore up any weaknesses in trigonometry or algebra found in entering students.
1.1 Introducing Calculus: Can change occur at an instant?	3.1	
1.2 Defining limits and using limit notation	2.1 (which includes the optional epsilon-delta definition)	
1.3 Estimating limit values from graphs	2.1	
1.4 Estimating limit values from tables	2.1 (e.g. Figure 2.1 & text)	
1.5 Determining limits using algebraic properties of limits	2.1 (e.g. see Theorem 1)	
1.6 Determining limits using algebraic manipulation	2.1 (for example, pp 62ff)	
1.7 Selecting procedures for determining limits	2.1 (examples and problems)	
1.8 Determining limits using the Squeeze theorem	2.1	Finley names this the "sandwich" theorem.

AP Syllabus Requirements	Covered in Finney Section	Additional Treatment Provided to Sharpen Focus
1.9 Connecting multiple representations of limits	Peripherally addressed in 2.1, e.g. Example 6 & Figure 2.4, Example 7, Figure 2.5, and Exercises 29-48 which involve multiple representations	*Course content will add additional emphasis addressing multiple representations of limits.*
1.10 Exploring types of discontinuities	2.1, 2.2, 2.3	
1.11 Defining continuity at a point	2.3 Definition	
1.12 Confirming continuity over an interval	2.3	
1.13 Removing discontinuities	2.3	
1.14 Connecting infinite limits and vertical asymptotes	2.2	
1.15 connect limits at infinity and horizontal asymptotes	2.2	
1.16. Working with the intermediate value theorem	2.3	
2.1 Defining average and instantaneous rates of change at a point	2.4	
2.2 Defining the derivative of a function and using derivative notation	3.1	
2.3. Estimating Derivatives of a function at a point	3.1	
2.4 Connecting differentiability and continuity: Determining when derivatives do and do not exist.	3.2	
2.5 Applying the Power Rule	3.3	
2.6. Derivative Rules: constant, sum, difference, and constant multiple	3.3	

AP Syllabus Requirements	Covered in Finney Section	Additional Treatment Provided to Sharpen Focus
2.7 Derivatives of cos x, sin x, e^x and ln x	3.5 Trigonometric 3.9 exponential and logarithmic	
2.8 The product rule	3.3	
2.9. The quotient rule	3.3	
2.10 Finding the derivatives of tangent, cotangent, secant, and/or cosecant functions	3.5	
3.1 The Chain Rule	3.6	
3.2 Implicit differentiation	3.7	
3.3 Differentiating inverse functions	3.8	
3.4 Differentiating inverse trigonometric functions	3.8	
3.5 Selecting procedures for calculating derivatives	Covered in review preparation for Test on Differentiation	
3.6. Calculating higher-order derivatives	3.3	
4.1 Interpreting the meaning of the derivative in context	3.4	
4.2 Straight-line motion: connecting position, velocity and acceleration	3.4	
4.3 Rates of change in applied contexts other than motion	3.4 (specifically addresses sensitivity to change in hybridization, and marginal cost and marginal revenue in Economics)	
4.4 Introduction to related rates	4.6	
4.5 Solving related rates problems	4.6	

AP Syllabus Requirements	Covered in Finney Section	Additional Treatment Provided to Sharpen Focus
4.6. Approximating values of a function using local linearity and linearization	4.5	
4.7 Using L'Hopitals Rule for determining limits of Indeterminate forms	8.2	*Finney delays this topic until much later in the course.*
5.1 Using the Mean Value Theorem	4.2	
5.2 Extreme Value Theorem, Global versus local extrema, and critical points	4.1	
5.3 Determining intervals on which a function is increasing or decreasing	4.2	
5.4 Using the first derivative test to determine relative (local) extrema	4.3	
5.5 Using the candidates test to determine absolute (global) extrema	4.3	
5.6. Determining concavity of functions over their domains	4.3	
5.7. Using the second derivative test to determine extrema	4.3	
5.8. Sketching graphs of functions and their derivatives	4.3	
5.9. Connecting a function, its first derivative, and its second derivative	4.3	
5.10 Introduction to optimization problems	4.4	
5.11 Solving optimization problems	4.4	
5.12 Exploring behaviors of implicit relations	3.7	

AP Syllabus Requirements	Covered in Finney Section	Additional Treatment Provided to Sharpen Focus
6.1 Exploring accumulations of change	5.1, then 7.1 ("Integral as Net Change")	In 5.1, Finney approaches the accumulation of change somewhat subtly with several pages of the discussion of the *area under various velocity curves*. [Thus finding the area under a rate-of-change function to find the accumulation of change.] This is later expanded somewhat in Finney's treatment of the Fundamental Theorem of Calculus. *However, to make this relationship much more explicit, and to conform to the nomenclature of the AP Course Requirements, we add additional explanation [such as "Interpreting the Behavior of Accumulation Functions Involving Area", Princeton Review AP Calculus BC Prep, 2023 p275ff] to make it more clear, so that the Essential Knowledge CHA-4.A.1, CHA-4.A.2, CH-4.A.3 and CHA-4.A.4 are met at this stage in the course.* Much later in Finney, in Section 7.1, the text more directly addresses "net change over time" (i.e., **accumulation**), although still not with the exact nomenclature required by the AP.
6.2 Approximating areas with Riemann Sums	5.2	
6.3 Riemann Sums, summation notation, and definite integral notation	5.2	

AP Syllabus Requirements	Covered in Finney Section	Additional Treatment Provided to Sharpen Focus
6.4 The fundamental theorem of calculus and accumulation functions	5.3 begins the discussion, culminating in 5.4	*Again Finney's treatment subtly introduces the accumulation of change including calculator graphical evaluation of definite integrals to an endpoint "x" but our course adds additional explicit explanation of the relationship.*
6.5 Interpreting the behavior of accumulation functions involving area	5.4	
6.6 Applying properties of definite integrals	5.3	
6.7 The fundamental theorem of calculus and definite integrals	5.4	
6.8 Finding antiderivatives and indefinite integrals: Basic rules and notation	5.4	With reference back to Corollary 3 introduced in section 4.2
6.9 Integrating using substitution	6.2	
6.10 Integrating Functions using long division and completing the square	Not explicitly covered in Finney	*We cover these techniques using the appropriate material from the instructor, Princeton Review and other sources. [such as Integrating Functions Using Long Division and Completing the Square, Princeton Review AP(r) Calculus BC Prep, 2023, p. 313ff]*
6.11. Integrating using Integration by Parts (BC)	6.3	
6.12 Using Linear partial fractions (BC)	8.4	*Finney covers this very, very briefly so we will provide somewhat larger coverage with material from the instructor, Princeton Review, and other sources.*
6.13. Evaluating improper integrals (BC)	8.4	

AP Syllabus Requirements	Covered in Finney Section	Additional Treatment Provided to Sharpen Focus
6.14 Selecting techniques for antidifferentiation	Covered as we go through antidifferentiation, with comparisons, and in section test preparation.	
7.1 Modeling Situations with differential equations	6.4	
7.2 Verifying solutions for differential equations	6.5	
7.3 Sketching slope fields	6.1	
7.4 Reasoning using slope fields	6.1, and then utilized in 6.2 and 6.3	
7.5 Approximating solutions using Euler's Method (BC)	6.1	
7.6 Finding general solutions using separation of variables	6.4	
7.7 Finding Particular solutions using Initial conditions and separation of variables	6.4	
7.8 Exponential models with differential equations	6.4	
7.9 Logistic models with differential equations (BC)	6.5	
8.1 Finding the average value of a function on an interval.	5.3	
8.2 Connecting position velocity and acceleration of functions using integrals	7.1	
8.3 Using accumulation functions and definite integrals in applied contexts	7.1	

AP Syllabus Requirements	Covered in Finney Section	Additional Treatment Provided to Sharpen Focus
8.4 Finding the area between curves expressed as functions of x	7.2	
8.5 Finding the area between curves expressed as functions of y	7..2	
8.6 Finding the area between curves that intersect at more than two points	7.2	
8.7 volumes with cross sections: squares and rectangles	7.3	
8.8 Volumes with cross sections: triangles and semicircles	7.3	
8.9 Volume with disc method: revolving around the x- or y-axis	7.3	
8.10 Volume with disc method: revolving around other axes	7.3	
8.11 Volume with washer method: revolving around the x- or y axis	7.3	
8.12 Volume with Washer method: revolving around other axes	7.3	
8.13 The arc length of smooth, planar curve and distance traveled (BC)	7.4	
9.1 Defining and differentiating parametric equations	10.1	
9.2 Second derivatives of parametric equations	10.1	
9.3 Finding arc lengths of curves given by parametric equations	10.1	
9.4 Defining and differentiating vector-valued functions	10.2	
9.5 Integrating vector valued functions	10.2	
9.6. Solving motion problems using parametric and vector-valued functions	10.1,10.2	

AP Syllabus Requirements	Covered in Finney Section	Additional Treatment Provided to Sharpen Focus
9.7 Defining polar coordinates and differentiating in polar form	10.3	
9.8 Finding the area of a polar region or the area bounded by a single polar curve	10.3	
9.9 Finding the area of the region bounded by two polar curves	10.3	
10.1 Defining convergent and divergent infinite series	9.1	
10.2 Working with geometric series	9.1	
10.3 the nth term test for divergence	9.1 ,9.4	
10.4 Integral Test for Convergence	9.5	
10.5 Harmonic series and p-series	9.5	
10.6 Comparison tests for convergence	9.5	
10.7 alternating series test for /convergence	9.5	
10.8. Ratio Test for convergence	9.4	
10.9 Determining absolute or conditional convergence	9.5	
10.10 Alternating series error bound	9.5	
10.11 Finding Taylor Polynomial Approximations of Functions	9.3	
10.12 Lagrange Error bound	9.3	
10.13 Radius and interval of convergence of power series	9.4	
10.14 Finding Taylor or Mclaurin Series for a function	9.2	
10.15 Representing functions as power series	9.1	

BIG IDEAS AND MATH PRACTICES

Likewise, our -------------AP Calculus class **teaching content** meets these Big Idea and Math Practices inclusion requirements as shown by the analysis of our course below:

CHAPTER	SECTION	BIG IDEAS	MATH PRACTICES
Chapter 1:	Prerequisites for Calculus		
	1.1 Lines		MP1, MP4
	1.2 Functions and Graphs		MP1, MP4
	1.3 Exponential functions		MP1, MP4
	1.4 Parametric equations		MP1, MP2, MP4
	1.5 Functions and Logarithms		MP1, MP4
	1.6. Trigonometric functions		MP1, MP2, MP4
Chapter 2	Limits and Continuity		
	2.1 Rates of Change and Limits	LIM	MP1, MP2
	2.2 Limits involving infinity	LIM	MP1, MP3
	2.3 Continuity	LIM	MP3
	2.4 Rates of change and tangent lines	LIM, CHA	MP2
Chapter 3	Derivatives		
	3.1 Derivative of a function	CHA	MP 1, MP4
	3.2 Differentiability	CHA, FUN	MP 2
	3.3 Rules for differentiation	FUN	MP 1
	3.4 Velocity and other rates of change	CHA	MP 1, 3
	3.5 Derivatives of trigonometric functions	FUN, LIM	MP 1
	3.6. Chain rule	FUN, CHA	MP 1
	37. Implicit differentiation	FUN	MP 1, MP3
	3.8 Derivatives of inverse trigonometric functions	FUN	MP1, MP3
	3.9 Derivatives of exponential and logarithmic functions	FUN	MP 1

CHAPTER	SECTION	BIG IDEAS	MATH PRACTICES
Chapter 4	Applications of derivatives		
	4.1 Extreme values of functions	FUN	MP3
	4.2 Mean value theorem	FUN	MP3
	4.3 Connecting f' and f'' with the graph of f	FUN	MP2
	4.4 Modeling and optimization	FUN	MP2, MP3
	4.5 Linearization and Newton's method		
	4.6. Related rates		
Chapter 5	The definite integral		
	5.1 Estimating with finite sums	CHA	MP4
	5.2 Definite integrals	LIM	MP2
	5.3 Definite integrals and anti derivatives	FUN	MP4
	5.4 Fundamental theorem of calculus	FUN	MP1, MP3
	5.5 Trapezoidal rule	LIM	MP2
Chapter 6	Differential equations and mathematical modeling		
	6.1 Slope fields and Euler's method	FUN	MP2, MP4
	6.2 Antidifferentiation by substitution	FUN	MP1
	6.3 Antidifferentiation by parts	FUN	MP1
	6.4 Exponential growth and decay	FUN	MP3
	6.5 Logistic growth	FUN	MP3
Chapter 7	Applications of definite integrals		
	7.1 Integral as net change	CHA	MP1
	7.2 Areas in the plane	CHA	MP1, MP4

CHAPTER	SECTION	BIG IDEAS	MATH PRACTICES
	7.3 Volumes	CHA	MP3
	7.4 Lengths of curves	CHA	MP3
	7.5 Applications from science and statistics	CHA	MP3
Chapter 8	Sequences, L'Hopital's Rule, and Improper Integrals		
	8.1 Sequences	LIM	MP3
	8.2 L'Hopital's Rule	LIM	MP3
	8.3 Relative Rates of Growth	CHA	MP1
	8.4 Improper integrals	LIM	MP1
Chapter 9	Infinite series		
	9.1 Power series	LIM	MP3
	9.2 Taylor Series	LIM	MP2, MP3
	9.3 Taylor's Theorem	LIM	MP2, MP3
	9.4 Radius of Convergence	LIM	MP3
	9.5 Testing Convergence at Endpoints	LIM	MP3
Chapter 10	Parametric, Vector and Polar Functions		
	10.1 Parametric functions	CHA	MP2
	10.2 Vectors in the plane	FUN	MP1
	10.3 Polar functions	FUN	MP2, MP3

Curricular Requirement 3

The course provides opportunities for students to develop the skills related to Mathematical Practice 1: Implementing Mathematical Processes, as outlined in the AP Course and Exam Description (CED).

Required Evidence ¨
The syllabus must include a description or copy of one or more lessons, activities, or assignments in which students use two or more skills under Mathematical Practice 1. The lessons, activities, or assignments must be described or identified so that the corresponding skill(s) are evident.

AND ¨

One of those lessons, activities, or assignments must incorporate the portion of Skill 1.E in which students apply appropriate mathematical rules or procedures **without technology**. [emphasis added] It is not necessary that the skills appear in a single lesson, activity, or assignment.

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EXAMPLES OF RELEVANT ASSIGNMENTS

Example	Homework Assignment	How Relevant Mathematical Practices Involved
1	Homework, Section 3.6 (Chain :Rule) includes problems 1-16, 26, 33, 41	Problems 1-8 require the use of provided substitutions, then the Chain Rule to obtain the final derivative using manual skills. **(MP 1.A, 1.B, 1.C, 1.D, 1.E without technology.)** Problems 9-11 require the student recognition of substitutions that can be made to recognize the problem as an *f o g* problem and then apply the Chain rule for manual differentiation to obtain the velocity of the objective, given a composite function for displacement. **(MP 1.A, 1.B, 1.C, 1.D, 1.E without technology.)** Problems 13-16 require the student to recognize substitutions that will allow a composite function and application of Chain Rule for manual differentiation. **(MP 1.A, 1.B, 1.C, 1.D, 1.E without technology.)** *For each of these problems, the assignment will be extended to require the student to state the appropriate derivative rule for each function. (MP 1.C)*
2	Homework, Section 5.1 (Definite Integral: Estimating with Finite Sums) problems	Problem 5 requires the student to sketch an area under a function and the x-axis, then partition it

	assigned 5a, 5B, 7 for LRAM, then #20 using calculator LRAM, both for N=8 and N=100.	into subintervals and manually calculate the approximation of the definite integral using the process of LRAM; problem 7 requires using a calculator (either an internal LRAM program or a special supplied PROGRAM that accomplishes LRAM) to calculate an approximation for various numbers of subintervals from 10 through 500. **(MP 1.A, 1.B, 1.C, 1.D)** Problem #20 Requires calculating an approximation of the volume of a solid hemisphere using partitions into multiple cylinders, using either the calculator LRAM or the instructor-developed PROGRAM for n=8 and n=100, **(MP 1.A, 1.B, 1.C, 1.D,)** *and is extended to additionally require the student to select and state the proper mathematical rule for calculating the correct hemispherical volume* **(MP 1E without technology)** *and to explain how the approximated value relates to the actual value.* **(MP 1.F)**

<u>Curricular Requirement 4</u>

The course provides opportunities for students to develop the skills related to Mathematical Practice 2: Connecting Representations, as outlined in the AP Course and Exam Description (CED).

<u>Required Evidence</u> ¨
The syllabus must include a description or copy of one or more lessons, activities, or assignments in which students work with multiple representations. Each of the four representations (analytical, numerical, graphical, and verbal) must be in at least one of the provided lessons, activities, or assignments. It is not necessary that all four representations appear in a single lesson, activity, or assignment.

AND ¨

There must be evidence of a connection between at least two different representations in at least one of the provided lessons, activities, or assignments aligned with Skills 2.C, 2.D, or 2.E. ¨

The lessons, activities, or assignments must be described or identified so that the skill(s) and representations are evident.

EXAMPLES OF RELEVANT ASSIGNMENTS

Example	Homework Assignment	How Relevant Mathematical Practices Involved
1	Finney Section 5.5 homework includes #13, 14, 15 (done using Trapezoidal Rule rather than Simpson's rule) among others.	**Analytical, Graphical Representations**. These problems present definite integrals as analytical representations and require the student to calculate approximations with implied graphical representations for the Trapezoidal Rule application, and also to relate them to the exact value of the same integrals computed explicitly. Thus it implicitly requires connection between two different representations. (2B, 2D) *For #13 and #14, the students will be asked to discuss the connections between the (a) analytical representation of the integral, (b) the implied graphical representation of the same integral and (c) the graphical Trapezoidal or Simpson's Rule utilized as an approximation. (2E)*
2	Finney Section 2.4 homework included #7, among several others.	**Graphical , Numerical, and Verbal Representations**. Problem #7 requires estimating the slopes of a presented graphical

		displacement versus position **graph**, arranging them into a **table**, determining the appropriate unit for these slopes and estimating the speed at a point. Thus it involves a connection between different representations. (2B, 2D) *This exercise is then extended to ask the students to discuss the connections between the different representations. (2E)*
3	Finney Section 5.5 Homework includes Problem #9 among several others	**Numerical Representation.** Problem #9 presents depth of a swimming pool at different positions as a numerical table (depth in feet for positions from 0 feet through 50 feet, every 5 feet) , and requires the use of the Trapezoidal Rule to calculate and estimate for the definite integral of the volume of the water. (2B, 2C) *The exercise will be extended for this problem to ask the student to discuss the connections between the numerical tabular data and the implied graphical representation. (2E)*

Curricular Requirement 5

The course provides opportunities for students to develop the skills related to Mathematical Practice 3: Justification, as outlined in the AP Course and Exam Description (CED).

Required Evidence ¨
The syllabus must include a description or copy of one or more lessons, activities, or assignments in which students use two or more skills under Mathematical Practice 3. The lessons, activities, or assignments must be described or identified so that the corresponding skill(s) are evident.

AND ¨

One of those skills must be 3.C, "Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied."

AND ¨

One of those skills must be either 3.E, "Provide reasons or rationales for solutions and conclusions," or 3.F, "Explain the meaning of mathematical solutions in context."

It is not necessary that the skills appear in a single lesson, activity, or assignment.

EXAMPLES OF RELEVANT ASSIGNMENTS

Example	Homework Assignment	How Relevant Mathematical Practices Involved
1	Assigned problems for Finney Section 4.2 included, among others, #1-#8	Each of these problems presented an analytic function and required the student to determine if the hypotheses of the Mean Value Theory, and if it did, to find the value of an internal point in the domain that satisfied the Mean Value Theory for the derivative (Theorem 3, p. 196). (3C, 3D)
2	Assigned problems for Finney Section 4.4 (Modeling and Optimization) included problems 1-4, 22, 23, among others. Their answers were defended in classroom discussion.	In each of these problems, students were required to find maxima or minima of real world values (e.g., in constructional dimensions, or economics) and were required to present evidence of application of one or more tests such as the First Derivative Test for Extrema, Second Derivative Test for Local Extrema in oral discussion to prove that they had met the requirements to find the

		requested maxima or minima of the problem. As real word problems, the meaning of the mathematical solutions had to be explained in context. (3B, 3C, 3D, 3E, 3F)

Curricular Requirement 6

The course provides opportunities for students to develop the skills related to Mathematical Practice 4: Communication and Notation, as outlined in the AP Course and Exam Description (CED).

Required Evidence ¨
The syllabus must include a description or copy of one or more lessons, activities, or assignments in which students are given the opportunity to communicate their understanding of calculus concepts, processes, or procedures using appropriate mathematical language. (Skill 4.A)

AND ¨

The syllabus must include a description or copy of one or more lessons, activities, or assignments in which students demonstrate notational fluency by either connecting different notations for the same concept or using appropriate mathematical notation in applying procedures. (Skill 4.C) ¨

The lessons, activities, or assignments must be described or identified so that the corresponding skill(s) are evident. It is not necessary that the skills appear in a single lesson, activity, or assignment.

EXAMPLES OF RELEVANT ASSIGNMENTS

Example	Homework Assignment	How Relevant Mathematical Practices Involved
1	Following the completion of Finney section 5.4 (Fundamental Theorem of Calculus), students are given a worksheet as described below. (At this point, students have been exposed to Riemann sums, the definite integral, the derivative, and its definitions as limits of difference quotient.) The worksheet has two columns where students must match equivalent forms of notation for calculus concepts, such as the limit of a Riemann sum with the number of subdivisions tending to infinity and the corresponding definite integral; limit of a different quotient and the corresponding derivative at a	The matching of equivalent forms of notations for calculus concepts allows the student to demonstrate the understanding and appropriate use of mathematical symbols and notation by recognizing different notations for the same concept. (4C)

	specific x-value; two equivalent forms of the limit of a difference quotient and similar notional representations. (4C) After students have worked individually to match equivalent forms of notation for calculus concepts, they share their answers to a partner, explaining their reasoning for each of the matches and then each pair orally explains to the class their reasoning for one match, with the instructor checking that they are using appropriate mathematical language. (4A)	The oral presentations allow the students the opportunity to communicate their understanding of calculus concepts using appropriate mathematical language. (4A).
2	Problems assigned for Finney 4.6 (Related Rates) included problems 1-7, 11, 19, 36-41	These problems involve the use of physical phenomena with related rates and thus the use of proper units. For example, Problem #11 involves inflating a helium spherical balloon at a volume rate of 100*pi ft^3/min and requires the student to calculate the rate of the increase of the radius (in feet) and the surface area (in ft2). (4B)

Curricular Requirement 7

Students have access to graphing calculators and opportunities to use them to solve problems and to explore and interpret calculus concepts.

Required Evidence
The syllabus includes a statement that each student has individual access to an approved graphing calculator.

AND

The syllabus must include a description or copy of one or more lessons, activities, or assignments in which students use graphing calculators to:

- graph functions
- solve equations
- perform numerical differentiation
- perform numerical integration
- explore or interpret calculus concepts

It is not necessary that the above requirements appear in a single lesson, activity, or assignment.

TECHNOLOGY
All students are expected to have a TI-84+CE calculator for their use in class and for homework assignments. For students that cannot afford this calculator, our school will loan a calculator to that student for the course.

The graphical calculator will be used very frequently in our class **(CR7)** and we will mentor students to help them become more adept in the use of this technology to help them understand the various concepts and to more efficiently process problems. Additionally, we will provide them with a basic understanding of how to create simple programs to carry out repetitive processes such as Riemann sums.

Throughout the course, students will be instructed in at least four functionalities allowed on the AP Exam with the graphical calculator including

- Plotting the graph of a function within an arbitrary viewing window
- Finding the zeros of a function (solving equations numerically)
- Numerically calculating the derivative of a function
- Numerically calculating the value of a definite integral (FNINT)

EXAMPLES OF RELEVANT ASSIGNMENTS

Example	Homework Assignment	Relevant computation
1	Problems assigned from Section 1.3 included #9 and #10 to be solved using both the TI-84 CE SOLVER and by graphing and using the TRACE function	These problems require the use of the TI-84 CE ability to both SOLVE equations within numerical bounds, and to graph functions and move a pointer ("trace") along the line of the graph. (**graph functions**; **solve equations**)
2	Problems assigned from Section 3.2 include #17, 18, 23, 24.	These problems require the use of TI-84 MATH nDeriv function as well as a recognition of where functions are actually differentiable, which can be assisted by graphing. (**numerical differentiation** and **graphing**)
4	Problems assigned from Section 5.2 (Definite Integrals) include #33, 35, and 36.	Problem 33 requires TI-84 MATH fnInt application; (**numerical integration**) Problems 35 and 36 require both graphing the functions and using MATH fnInt to help evaluate the "areas." (**graph functions** and **numerical integration**).
3	Problems assigned in Section 5.5 included an additional assignment to modify a provided TI-84 CE "program" that evaluates LRAMs (Left-hand endpoint rectangular approximation method), to make it evaluate using the Trapezoidal Rule and then use it to evaluate problems #13 and #14 with N=100 and compare to the exact value of the integral. *This exercise is then extended by having the students evaluate with smaller values of N (more coarse approximation) and intermediate and larger values of of N (finer approximations to the curve).*	This assignment added some programming expertise to the student, and allowed them to further explore and interpret the definite integral. (**numerical integration**; **explore and interpret calculus concepts**) *The process of editing and rewriting the software allows the student to explore the actual procedures of the Trapezoidal Rule's approximation of the definite integral and additionally address the concept of the definite integral as an accumulation. The application of the newly created software to evaluate approximations of the definite integral with smaller, intermediate and larger values of N provides the student the opportunity to observe the apparent limit of the Trapezoidal Rule as it draws ever closer to the exact value of the integral.*

Curricular Requirement 8

The course provides opportunities for students to use calculus to solve real-world problems.

Required Evidence

The syllabus must provide a description of one or more lessons, activities, or assignments requiring students to apply their knowledge of AP Calculus concepts to solve real-world problems.

EXAMPLES OF RELEVANT ASSIGNMENTS

Example	Homework Assignment	Real World Problems (Explanation)
1	Problems assigned for Section 4.6 included among others, problems 17, 18, 19.	Problem 17 involves draining a conical concrete reservoir of water of given dimensions, at a constant volumetric rate, and finding the rate at which the water level is falling at a specific remaining height, and what is the rate of the change in the radius of the surface area. Problem 18 involves draining a hemispherical bowl of given dimensions, at a constant volumetric rate and finding the rate of water level change, and rate of the radius changing at a given remaining water level. Problem 19 is the classical sliding ladder problem with a base slipping out at given rate and requesting the student to find the rate of the top dropping.
2	Review problems assigned for Chapter 4 (Applications of Derivatives) included problem 62 among others..	Problem 62 involves water draining from a conical tank of given dimensions at a constant volumetric rate, and requires the student to determine the relationship between the remaining height of water and the remaining radius of water and solve for the rate of water level decline at a certain remaining height.

Classical Christian Science Teaching

We teach at a classical Christian school. Traditionally the theory of classical Christian education revolves around a trivium of Grammar, Logic, and Rhetoric. These categorizations apply well to many of the subjects of the humanities. It has been questioned at times, and formally discussed how much difficulty there is in applying these to the Sciences and Mathematics. James W. Seidel has provided an insightful evaluation of past educational theories from secular, Christian, classical, and classical Christian schools of thought, and synthesized proposals for the basis for modern classical Christian mathematical education.[3]

Seidel provides the background for classical Christian mathematical education:

> The starting point for understanding the classical side of CCE mathematics comes, as it did more generally for Sayers in her original essay, from examining medieval classical education. In that approach, the quadrivium contained the four mathematical arts. These four arts constituted the primary core mathematical knowledge in their day. Students schooled in the quadrivium received a comprehensive exposure to the main fields of elementary mathematics at the time. Applying the same principle, therefore, modern CCE mathematics education should aim to teach all developmentally-appropriate mathematical knowledge available to us today [4]

Seidel argues that Classical Christian Mathematics has basic pillars of instruction:

- The first goal is to develop capable Christian disciples, and a knowledge of mathematics is a requirement for success.
- Secondly, classical Christian mathematics should develop wonder in the student, as they come to recognize the beauty of mathematics, the mathematical features of our physical world, and gain in their understanding of the grandeur of God.[5]
- As a result of the second goal, real-world examples from our Creation should be solidly included within a classical Christian mathematical education -- and in our Calculus course they are in abundance!
- Fourthly, classical Christian mathematical education should maintain a balance between application and theory -- and the well trained Christian disciple should be able to use mathematics accurately to guide public policy decisions, including providing justifications, and

3 Seidel JW. Mathematics: Giving Classical, Christian Education Its Voice. (2011) Dordt University, Master of Education Program Thesis. Accessed Nov 11 2022 at https://digitalcollections.dordt.edu/cgi/viewcontent.cgi?article=1034&context=med_theses

4 Mathematics: Giving Classical Christian Education Its Voice, p 38.

5 Just as I have argued for classical Christian science education, to truly understand the scope and grandeur of the glory of God, a considerable education in Mathematics is certainly required. The mathematician can see aspects of the glory of God that are hidden from others' eyes.

explanations to help the citizenry understand the truth from mathematics about crucial decisions.

- Classical Christian mathematical education includes a "grammar" of theorems & definitions that come from past experts and understanding the historical acquisition of this knowledge can also be an important part of the mathematical education. It should develop the logical reasoning that properly applies these theorems to mathematical problems to produce solid conclusions, which can be tightly justified and explained to others. Thus communication (rhetoric) is an important part of a classical Christian mathematical education.

It is noteworthy how much correlation there is between the Curricular Requirements of the AP College Board, and these tenets of classical Christian mathematics education! The careful application of theorems and definitions required by the College Board is clearly part of classical Christian mathematics. The emphasis of the College Board on real-world problems is in direct agreement with the classical Christian mathematics goal of not only understanding the glory of the Creation but also being a good disciple able to properly apply mathematics to the societal problems of the day, reaching correct mathematical conclusions about the advisability of different solutions, and providing lucid communications that persuade others from solid reasoning

Our school's classical Christian teaching of Calculus will thus both meet the requirements of the AP College Board and the tenets of our understanding of proper teaching of mathematics.

RECOMMENDATIONS

Students are encouraged to remember me when they need to file letters of recommendation or have character references. With my background I have dealt with tens of thousands of patients and families and many many other professionals. My friends are often leaders in law or public service, or education. It will do our students good to get to know quality examples of Christian Leaders in our community who have stood the test of time and can accurately evaluate the character and performance of aspiring young people who have an entire life ahead them and important choices to make. Everyone deserves a good recommendation for the effort they have put forward, and everyone has their own set of God-given gifts -- we are not the same! Finding the niche for which God developed each and every one of us is part of the Christian walk, and if I can help a student with that, it's great.

By signing below, you are signifying that you understand and agree to the above terms of education of AP Calculus

______________________________ __________________________

Student Signature Date

______________________________ __________________________

Parent(s) Signature Date